PROPOSAL OF AN ANALYTICAL TECHNIQUE USING CIRCULARLY POLARIZED WAVES AND ITS APPLICATION

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A new analytical technique is constructed by using circularly polarized waves. And further, it is applied to calculate and interpret the polarization characteristics of optical fiber.

1 Introduction

The polarization characteristics of a single mode fiber is measured by degree of polarization and polarizing angle. The degree of polarization and polarizing angle are decided by amplitude and phase of a propagating waves. Therefore, it is useful for practical investigations of fiber to gain a simple relation between measuring parameters(degree of polarization and polarizing angle) and electrical parameters(amplitude and phase of a wave).

The author has introduced the right-handed and left-handed circularly polarized waves of a step-index optical fiber[1]. A propagating wave of a single mode fiber can be always expanded by two circularly polarized waves. When this expansion technique is applied to obtain the relation between measuring parameters and electrical parameters, the relation becomes very simple comparing with former results [2] ,[3].

The expansion technique is also applied to analyze the polarization characteristics of a twisted elliptical core fiber. Then, some properties are revealed.

By the way, it is experimentally known that in a twisted fiber there exist rapidly increasing and decreasing phenomena of polarizing angle with respect to the increasing twist angle[4]. However, the origin from which these phenomena come has not yet been understood. An interpretation of these phenomena is brought out by using the above mentioned relations.

2 Circularly polarized waves in a circular core fiber

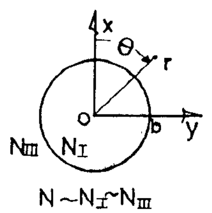

Fig. 1 Circular core fiber cross-section and coordinate systems

The fiber cross-section and coordinate systems are shown in Fig.1. The author has introduced the right-handed and left-handed circularly polarized waves belonging to the HE_{11} mode in a following forms:

(left-handed circularly polarized wave)

$$\mathbb{E}^l = k\,N\,u\,J_0(u\,r)\,(-j\,\mathring{i}_x + \mathring{i}_y) + u^2 J_1(u\,r)\,e^{j\theta}\,\mathring{i}_z \quad e^{-j\,h\,z + j\omega t} \tag{1a}$$

$$\mathbb{H}^l = \mathbb{E}^l \,/\, (\,j\,\eta\,) \qquad \text{in a core,}$$

(right-handed circularly polarized wave)

$$\mathbb{E}^r = k\,N\,u\,J_0(u\,r)\,(-j\,\mathring{i}_x - \mathring{i}_y) + u^2 J_1(u\,r)\,e^{j\theta}\,\mathring{i}_z \quad e^{-j\,h\,z + j\omega t} \tag{1b}$$

$$\mathbb{H}^r = -\,\mathbb{E}^r \,/\, (\,j\,\eta\,) \qquad \text{in a cladding,}$$

where u and h are the radial propagation constant and the propagation constant, respectively, $\eta\,(=\sqrt{\mu_0/\varepsilon_0}/N)$ and b denote the wave impedance and the core radius, respectively, $J_n(z)$ and $K_n(z)$ represent the Bessel and modified Bessel functions, respectively, $(\mathring{i}_x, \mathring{i}_y, \mathring{i}_z)$ mean the unit vectors of the Cartesian coordinate. (1c)

3 The relation between measuring and electrical parameters

The electric field of the HE_{11} mode($\mathbb{E}$) is always expanded by using the circularly polarized waves as follows

$$\mathbb{E} = A\;\mathbb{E}^l + B\;\mathbb{E}^r. \tag{2}$$

Put A and B in polar forms

$$A = |A|\,e^{j\theta_A}\,,\quad B = |B|\,e^{j\theta_B}. \tag{3}$$

Substituting (2) into the Maxwell's equation and after some calculations, the following relation results between the polarizing angle(ψ) and the phase of amplitude(θ_A , θ_B)

$$\psi = (\,\theta_B - \theta_A)/2 \pm n\pi/2, \quad (n=0,1,2,\ldots.) \tag{4}$$

where even n and odd n correspond to the main and minor axes of an elliptically polarized wave, respectively [1]. ψ is measured from the x-axis.

On the other hand, the main axis(E_ξ) and the minor axis(E_η) of an elliptically polarized wave are expressed by $|A|$ and $|B|$ in the next form[1]

$$E_\xi = |A| + |B|\,,\quad E_\eta = \big|\,|A| - |B|\,\big|. \tag{5}$$

The degree of polarization(I) is defined by

$$I = (\,E_\xi^2 - E_\eta^2)\,/\,(\,E_\xi^2 + E_\eta^2). \tag{6}$$

Substituting (5) into (6), we obtain

$$I = 2\,|A|\cdot|B|/(\,|A|^2 + |B|^2\,). \tag{7}$$

The equation (4), (5) and (7) are the relations to be expected. These relations are very simple comparing with the forms obtained by the expansion technique using linearly polarized waves[5].

4 Polarization characteristics of a twisted elliptical core fiber

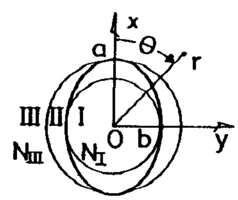

Fig. 2 Elliptical core fiber cross-section and coordinate systems

The field in a twisted elliptical core fiber is assumed to be approximately expanded in the form shown in (2). Figure 2 shows the cross-section of a twisted elliptical core fiber and coordinate systems. Substituting (2) into the Maxwell's equation and after some calculations, the following simultaneous differential equation is obtained[6]:

$$\frac{\partial A}{\partial z} = -j(\alpha_1 + \alpha_2)A - j\beta e^{-2j\phi_0 z}B,$$
$$\frac{\partial B}{\partial z} = -j(\alpha_1 - \alpha_2)B - j\beta e^{2j\phi_0 z}A, \tag{8}$$

where $\alpha_1 = \dfrac{1.6(1-D)\,U^2 K_0^2(W)}{\pi b K N D^2\, K_1^2(W)}$, $\beta = \dfrac{0.8(1-D)\,U^2 W^2}{2\pi b\, K^3 N^3 D^2}$

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$$\alpha_2 = G C \phi_o / N \tag{9}$$

In equation (9), a and b are the radii of the main and minor axes of core, respectively, G, C and N denote the rigidity coefficient, the photoelastic constant and the refractive index, respectively, ϕ_o and k represent the twist angle per unit length and the wave number of free space, respectively, and $D=b/a$, $U=u\,b$, $W=w\,b$ and $K=k\,b$. The others are common to (1c).

The solution of (8) is expressed as follows:

$$A = e^{-j(\alpha_1+\phi_o)z}\left[A(0)\cos(\kappa z) - j\,\frac{(\alpha_2-\phi_o)A(0)+\beta B(0)}{\kappa}\cdot\sin(\kappa z)\right]$$

$$B = e^{-j(\alpha_1-\phi_o)z}\left[B(0)\cos(\kappa z) + j\,\frac{(\alpha_2-\phi_o)B(0)-\beta A(0)}{\kappa}\cdot\sin(\kappa z)\right] \tag{10}$$

where $\kappa = ((\alpha_2-\phi_o)^2+\beta^2)^{\frac{1}{2}}$, $A(0)=A|_{z=0}$, $B(0)=B|_{z=0}$.

When the core becomes circular i.e. $\alpha_1=\beta=0$, equation (10) reduces to

$$A = A(0)\ e^{-j\alpha_2\cdot z}, \quad B = B(0)\ e^{j\alpha_2\cdot z}. \tag{11}$$

This result insists that the circularly polarized waves are the eigen-modes. According to (4), we obtain the polarizing angle rotation

$$\Theta_L(\text{rad}) = 2\,\alpha_2\,L/2 = G\,C\,\phi_o\,L/\,N, \tag{12}$$

where L means fiber length. On the other hand, as the twist angle becomes very large i.e. $\beta/\kappa \to 0$, equation (10) is reduced to

$$A = A(0)\ e^{-j\alpha_2\cdot z}, \quad B = B(0)\ e^{j\alpha_2\cdot z}. \tag{13}$$

which is the same as (11). This fact shows that a well twisted optical fiber has a polarization-maintaining property and is suitable to circularly polarized wave transmission.

Imposing the condition shown in table I and $A(0)=B(0)=E_i$ on (10), numerical results are calculated.

Table I Parameters used in numerical calculations

V	2.26	L	20.4(cm)
Δ	0.0032	βL	1.204
N	1.457	G·C	0.4
b	1.95μm	λ	0.71 μm
b/a	0.8		

V: Normalized Frequency
$\Delta = 1 - N_{III}/N_I$
λ: Wave Length
L: Fiber Length

Some examples of polarization characteristics are illustrated in Fig.3 and Fig.4. Figure 3 shows the case for the incident polarizing angle $\psi_i = 4.5^o$ and Fig.4 for $\psi_i = 6.0^o$. ψ_i is measured from the x-axis. A solid line and a dotted line correspond to E_ξ and E_η, respectively. Rapidly increasing and decreasing phenomena of the polarizing angle(ψ) are found at about $\phi_o L = 100^o$ in lower parts of Fig.3 and Fig.4. On the other hand, in upper parts of Fig.3 and Fig.4, the relation $E_\xi \cong E_\eta$ is obtained near this point, so that the wave is expected to be circularly polarized.

5 An interpretation of rapidly increasing and decreasing phenomena of polarizing angle

Putting $A(0)=B(0)=1$, let the other parameters be the same shown in table I. Then we express A and B in the form shown in (3). The loci of A and B are illustrated in the upper parts of Fig.5 and Fig.6 as a function of twist angle($\phi_o L$) denoted by dotts. The calculated region of the twist angle is $[-100^o, 800^o]$ for A and $[-200^o, 800^o]$ for B. The locus of A rotates clockwise according to the increasing twist angle and the locus of B counter-clockwise. The locus of A near about $\phi_o L = 100^o$ pass through close to an upper part or a lower part of the coordinate origin. The phase Θ_A

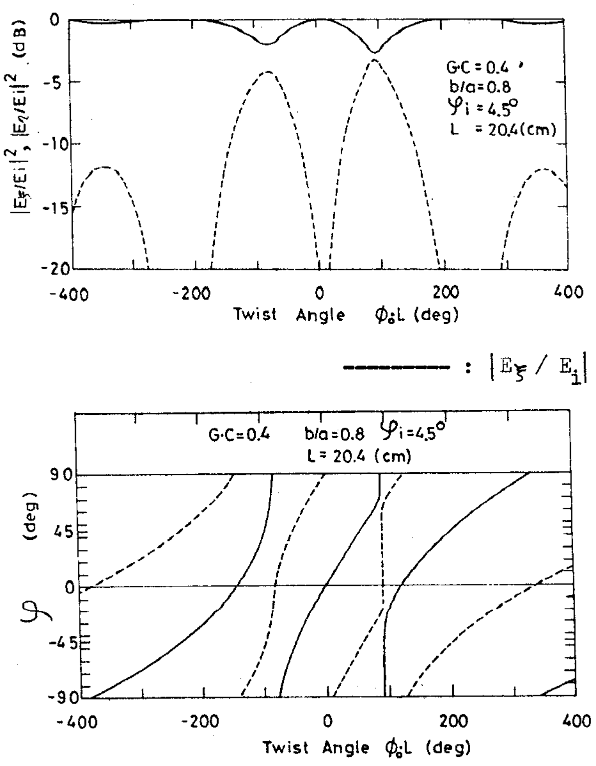

Fig. 3 E_ξ , E_η and ψ of an elliptically polarized wave vs. twist angle ($\phi_o \cdot L$)
$\psi_i = 4.5^o$

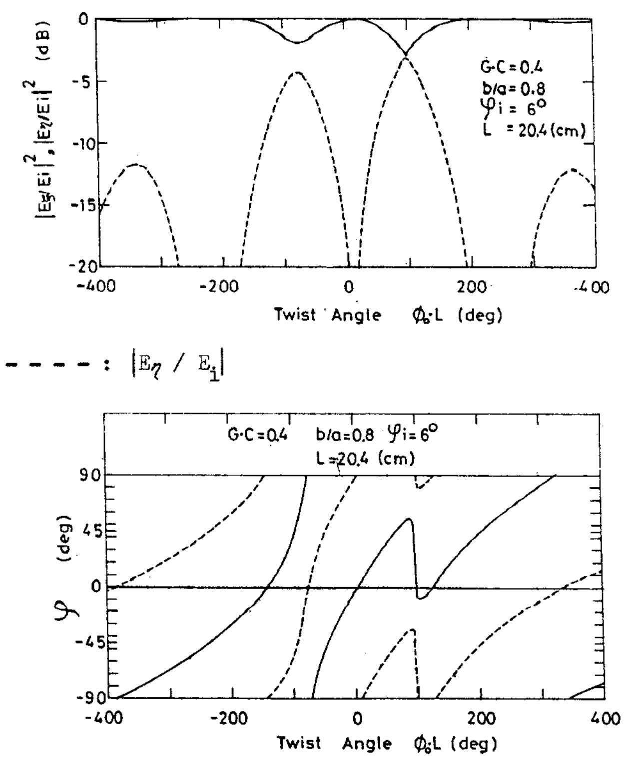

Fig. 4 E_ξ , E_η and ψ of an elliptically polarized wave vs. twist angle ($\phi_o \cdot L$)
$\psi_i = 6.0^o$

θ_B are shown in the lower parts of Fig.5 and Fig.6. θ_A is rapidly decreasing in Fig.5 and increasing in Fig.6. On the other hand, θ_B is stationary near this point in both figures. Therefore, we have a suddenly increasing polarizing angle in Fig.5 and a rapidly decreasing polarizing angle in Fig.6 according to (4). These results correspond to the phenomena pointed out in Fig.3 and Fig.4. It becomes clear that the origin of the above phnomena is whether the loci of A and B in the complex-plane pass through close to upper or lower parts of the coordinate origin.

6 Conclusion

The analytical technique using circularly polarized waves are proposed. Relation between measuring and electrical parameters become simple in this technique, which are expressed in (4), (5) and (7).

In a twisted circular core fiber and a well twisted optical fiber, the polarizing angle rotates at the rate shown in (12), which is proportional to the twist angle so that the fibers act as optical active media. A well twisted optical fiber has a polarization-maintaining property and is suitable to a circularly polarized transmission.

Using the relation shown in (4), an interpretation is made for the rapidly increasing and decreasing phenomena of polarizing angle occuring near the condition on which a circularly polarized wave takes place. It appears that the origin of these phenomena is whether the loci of A and B pass through close to upper or lower parts of the coordinate origin in the complex-plane

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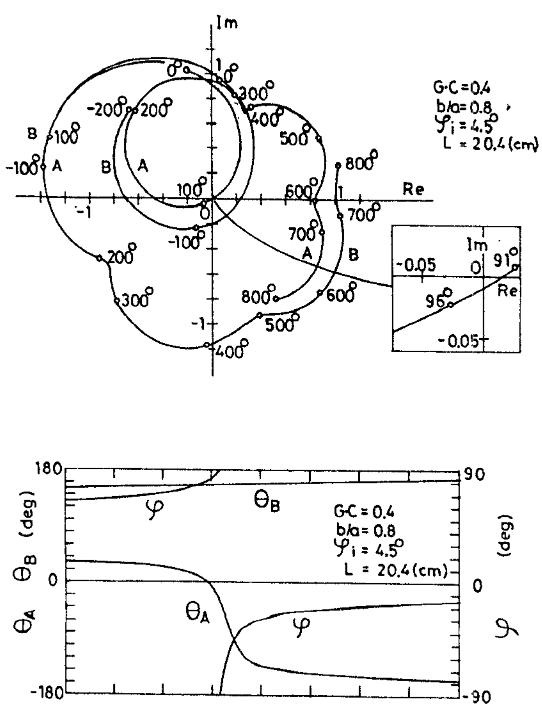

Fig. 5 Upper part : loci of A and B vs. twist angle
Lower part : phase angle (θ_A, θ_B) and φ vs. twist angle

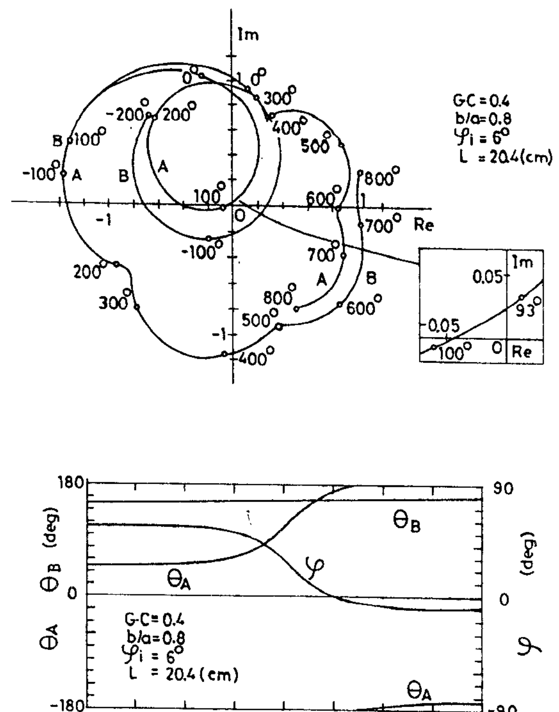

Fig. 6 Upper part : loci of A and B vs. twist angle
Lower part : phase angle (θ_A, θ_B) and φ vs. twist angle